

BFKL resummation effects in the $\gamma^*\gamma^*$ total hadronic cross section

F. Caporale^{1†}, D.Yu. Ivanov^{2¶} and A. Papa^{1‡}

¹ *Dipartimento di Fisica, Università della Calabria,
Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza,
Arcavacata di Rende, I-87036 Cosenza, Italy*

² *Sobolev Institute of Mathematics,
630090 Novosibirsk, Russia*

Abstract

We study in the BFKL approach the total hadronic cross section for the collision of two virtual photons for energies in the range of LEP2 and in the range of future linear colliders. The BFKL resummation is done at the next-to-leading order in the BFKL Green's function; photon impact factors are taken instead at the leading order, but with the inclusion of the subleading terms required by invariance under changes of the renormalization scale and of the BFKL scale s_0 . We compare our results with previous estimates based on a similar kind of approximation.

[†]*e-mail address:* caporale@cs.infn.it

[¶]*e-mail address:* d-ivanov@math.nsc.ru

[‡]*e-mail address:* papa@cs.infn.it

1 Introduction

The total hadronic cross section for the collision of two off-shell photons with large virtualities is a fundamental observable, since, similarly to the process of e^+e^- annihilation into hadrons, is fully under control of perturbative QCD. In fixed order calculations, the dominant contribution at low energies comes from the quark box, calculated at the leading-order (LO) in Ref. [1] (see also Ref. [2]) and at the next-to-LO (NLO) in Ref. [3]. In Ref. [4] the resummation of double logs appearing in the NLO corrections to the quark box has been also studied. At higher energies the diagrams with gluon exchange in the t -channel become more important since they have different power asymptotics for $s \rightarrow \infty$ in comparison to the t -channel quark exchange; at higher orders such contributions contain powers of single logarithms of energy which can be resummed in the frame of the BFKL approach.

It is widely believed that the $\gamma^*\gamma^*$ total cross section is the best place for the possible manifestation of the BFKL dynamics [5] at the energies of future linear colliders (for a review, see Ref. [6]). For this reason, many papers [7] have considered the inclusion of the BFKL resummation of leading energy logarithms. In a remarkable paper [8] (see also Ref. [9]), BFKL resummation effects have been taken into account also at the subleading order and evidence has been presented that the appearance of BFKL dynamics is compatible with experimental data already at the energies of LEP2 [10, 11].

In the BFKL approach, both in the leading logarithmic approximation (LLA), which means resummation of leading energy logarithms, all terms $(\alpha_s \ln(s))^n$, and in the next-to-leading approximation (NLA), which means resummation of all terms $\alpha_s(\alpha_s \ln(s))^n$, the imaginary part of the amplitude for a large- s hard collision process can be written as the convolution of the Green's function of two interacting Reggeized gluons with the impact factors of the colliding particles (see, for example, Fig. 1).

The Green's function is determined through the BFKL equation and is process-independent. The NLO kernel of the BFKL equation for singlet color representation in the t -channel and for forward scattering, relevant for the determination of a total cross section, has been achieved in Refs. [12], after the long program of calculation of the NLO corrections [13] (for a review, see Ref. [14]).

The other essential ingredient to build up the $\gamma^*\gamma^*$ total cross section is the impact factor for the virtual photon to virtual photon transition. Its calculation in the NLO is rather complicated and has been completed only after year-long efforts [15]. The result, obtained in the momentum representation, is known to a large extent in numerical form. After gathering the NLO virtual photon impact factors with the NLO BFKL Green's function, the prediction for the energy dependence of the NLA $\gamma^*\gamma^*$ total cross section will become available. This remaining step is, however, rather difficult, so it may be interesting in the meanwhile to get an estimate of NLA BFKL effects, using an approximated procedure and possibly refining previous analysis of the same kind.

In this paper we estimate the energy dependence of the $\gamma^*\gamma^*$ total cross total hadronic in an energy range which covers LEP2 and future linear colliders. The procedure we follow is approximate, since we use the singlet forward NLO BFKL Green's function together with

forward $\gamma^* \rightarrow \gamma^*$ impact factors *at the leading order*. However, in the impact factors we include the subleading terms required by the invariance of the full amplitude at the NLA under change of the renormalization scale and of the energy scale s_0 entering the BFKL approach. The neglect of other subleading corrections to the impact factor certainly affects the low energy behavior of the cross section, but should not spoil the high energy regime. A more detailed discussion on this point will be presented later on.

The calculation goes along the same steps as for the amplitude $\gamma^*\gamma^* \rightarrow VV$, with $V = \rho^0, \omega, \phi$ a light neutral vector meson. In that case, however, the relevant impact factor, namely the $\gamma^* \rightarrow V$ impact factor, was available in closed analytic form in the NLO, up to contributions suppressed as inverse powers of the photon virtuality [16]. Therefore, the amplitude could be evaluated fully in the NLA [17, 18], previous estimations being based on fixed perturbative order calculations [19] and on partial inclusion of NLA BFKL effects [20]¹. As in Refs. [17, 18], the convolution between the impact factors of the colliding photons, taken with equal virtualities, and the BFKL Green's function is performed in the space conjugated to the transverse momentum space, namely in the so-called ν -space or, equivalently, through the spectral decomposition on the eigenfunctions of the LO BFKL kernel. Similarly to Refs. [17, 18] the large NLA corrections are handled by the adoption of suitable optimization methods of the perturbative series; in particular, we used the principle of minimal sensitivity (PMS) [25] and the the Brodsky-Lepage-Mackenzie (BLM) method [26].

The approximation of using LO impact factors in combination with NLO BFKL Green's function is not new. It has been exploited already in in Ref. [8] for the $\gamma^*\gamma^*$ total cross section, in Ref. [20] for the $\gamma^*\gamma^* \rightarrow VV$ amplitude, in Ref. [21] for the production of Mueller-Navelet jets at hadron colliders and in Refs. [22, 23] for the production of forward jets in deep-inelastic-scattering. In comparison with Ref. [8], in the present paper, the elements of novelty are the following:

- the optimization procedures to stabilize the perturbative series are performed on the amplitude itself and not on the NLO Pomeron intercept; we believe that this is more natural since a perturbative intercept is not a physical quantity;
- the impact factors, although taken at the LO, contain the appropriate NLO terms, so that the dependence on the energy scales entering the process (the renormalization scale μ_R and the parameter s_0 introduced in the BFKL approach) is pushed to the next-to-NLA; this makes the effect of s_0 on the numerical result less pronounced than in Ref. [8]; moreover, in our approach the value of s_0 (as well as that of μ_R) is determined by the optimization procedure and is not a free parameter;
- two optimization methods are used, thus having a control of systematic effects at work.

The paper is organized as follows: in the next Section we give the expression of the cross section; in Section 3 we present the numerical results; in Section 4 we draw our conclusions.

¹See also Ref. [24] for an analysis of the QCD factorization properties of this amplitude.

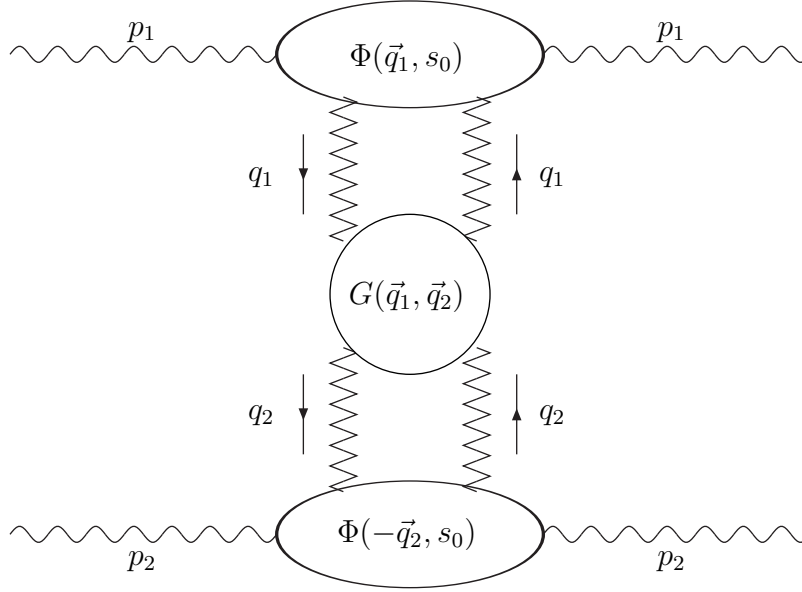


Figure 1: Schematic representation of the elastic amplitude for the $\gamma^*(p_1) \gamma^*(p_2)$ forward scattering.

2 The $\gamma^* \gamma^*$ total cross section

The total hadronic cross section of two unpolarized photons with virtualities Q_1 and Q_2 can be obtained from the imaginary part of the forward amplitude (see Fig 1) and within LO BFKL is given by the following expression (see, for instance, Ref. [8]):

$$\sigma_{tot}^{\gamma^* \gamma^*}(s, Q_1, Q_2) = \sum_{i,k=T,L} \frac{1}{(2\pi)^2 Q_1 Q_2} \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} F_i(\nu) F_k(-\nu) \left(\frac{s}{s_0} \right)^{\bar{\alpha}_s \chi(\nu)}, \quad (1)$$

where $\bar{\alpha}_s \equiv \alpha_s N_c / \pi$, $\chi(\nu)$ is the so-called characteristic BFKL function,

$$\chi(\nu) = 2\psi(1) - \psi\left(\frac{1}{2} + i\nu\right) - \psi\left(\frac{1}{2} - i\nu\right), \quad (2)$$

and

$$F_T(\nu) = F_T(-\nu) = \alpha \alpha_s \left(\sum_q e_q^2 \right) \frac{\pi \left(\frac{3}{2} - i\nu \right) \left(\frac{3}{2} + i\nu \right) \Gamma\left(\frac{1}{2} - i\nu\right)^2 \Gamma\left(\frac{1}{2} + i\nu\right)^2}{2 \Gamma(2 - i\nu) \Gamma(2 + i\nu)}, \quad (3)$$

$$F_L(\nu) = F_L(-\nu) = \alpha \alpha_s \left(\sum_q e_q^2 \right) \pi \frac{\Gamma\left(\frac{3}{2} - i\nu\right) \Gamma\left(\frac{3}{2} + i\nu\right) \Gamma\left(\frac{1}{2} - i\nu\right) \Gamma\left(\frac{1}{2} + i\nu\right)}{\Gamma(2 - i\nu) \Gamma(2 + i\nu)} \quad (4)$$

are the LO impact factors for transverse and longitudinal polarizations, respectively. In the previous equations, α is the electromagnetic coupling constant, the summation extends over all active quarks (taken massless) and e_q is the quark electric charge in units of the electron charge. In the expression (1) for the LO BFKL cross section the argument of the strong and electromagnetic coupling constants and the value of the scale s_0 are not fixed.

Following the procedure of Refs. [17], it is possible to write down the cross section with the inclusion of NLO corrections in the Green's function only, while keeping the impact factors at the LO:

$$\sigma_{tot}^{\gamma^*\gamma^*}(s, Q_1, Q_2) = \frac{1}{(2\pi)^2 Q_1 Q_2} \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \left(\frac{s}{s_0} \right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)} \sum_{i,k=T,L} F_i(\nu) F_k(-\nu) \quad (5)$$

$$\times \left\{ 1 + \bar{\alpha}_s^2(\mu_R) \ln \left(\frac{s}{s_0} \right) \left[\bar{\chi}(\nu) + \frac{\beta_0}{8N_c} \chi(\nu) \left(-\chi(\nu) + \frac{10}{3} + 2 \ln \frac{\mu_R^2}{Q_1 Q_2} \right) \right] \right\},$$

where

$$\begin{aligned} \bar{\chi}(\nu) = & -\frac{1}{4} \left[\frac{\pi^2 - 4}{3} \chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} \right. \\ & \left. + \frac{\pi^2 \sinh(\pi\nu)}{2\nu \cosh^2(\pi\nu)} \left(3 + \left(1 + \frac{n_f}{N_c} \right) \frac{11 + 12\nu^2}{16(1 + \nu^2)} \right) + 4\phi(\nu) \right], \end{aligned} \quad (6)$$

$$\phi(\nu) = 2 \int_0^1 dx \frac{\cos(\nu \ln(x))}{(1+x)\sqrt{x}} \left[\frac{\pi^2}{6} - \text{Li}_2(x) \right], \quad \text{Li}_2(x) = - \int_0^x dt \frac{\ln(1-t)}{t}. \quad (7)$$

In fact, the requirement of invariance of the amplitude at the NLA under renormalization group transformation and under change of the energy scale s_0 allows to fix the μ_R - and s_0 -dependent terms in the NLO impact factors, so that we can get the following expression for the total cross section, given by

$$\begin{aligned} \sigma_{tot}^{\gamma^*\gamma^*}(s, Q_1, Q_2) = & \frac{1}{(2\pi)^2 Q_1 Q_2} \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \left(\frac{s}{s_0} \right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)} \sum_{i,k=T,L} F_i(\nu) F_k(-\nu) \\ & \times \left\{ 1 + \bar{\alpha}_s(\mu_R) A(s_0) + \bar{\alpha}_s(\mu_R) B(\mu_R) + \bar{\alpha}_s^2(\mu_R) \ln \left(\frac{s}{s_0} \right) \left[\bar{\chi}(\nu) \right. \right. \\ & \left. \left. + \frac{\beta_0}{8N_c} \chi(\nu) \left(-\chi(\nu) + \frac{10}{3} + 2 \ln \frac{\mu_R^2}{Q_1 Q_2} \right) \right] \right\}, \end{aligned} \quad (8)$$

with

$$A(s_0) = \chi(\nu) \ln \frac{s_0}{Q_1 Q_2}, \quad B(\mu_R) = \frac{\beta_0}{2N_c} \ln \frac{\mu_R^2}{Q_1 Q_2}. \quad (9)$$

The above expression for the cross section can be conveniently represented as a series,

$$Q_1 Q_2 \sigma_{tot}^{\gamma^*\gamma^*} = \frac{1}{(2\pi)^2} \left\{ b_0 + \sum_{n=1}^{\infty} \bar{\alpha}_s(\mu_R)^n b_n \left[\ln^n \frac{s}{s_0} + d_n(s_0, \mu_R) \ln^{n-1} \frac{s}{s_0} \right] \right\}, \quad (10)$$

with coefficients

$$b_n = \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \sum_{i,k=T,L} F_i(\nu) F_k(-\nu) \frac{\chi^n(\nu)}{n!}, \quad (11)$$

determined by the kernel and the impact factors in LLA, and

$$d_n = n \ln \frac{s_0}{Q_1 Q_2} + \frac{\beta_0}{4N_c} \left[\frac{b_{n-1}}{b_n} \left((n+1) \ln \frac{\mu_R^2}{Q_1 Q_2} + \frac{5}{3}(n-1) \right) - \frac{n(n-1)}{2} \right] \\ + \frac{1}{b_n} \int_{-\infty}^{+\infty} d\nu \left(\frac{Q_1^2}{Q_2^2} \right)^{i\nu} \sum_{i,k=T,L} F_i(\nu) F_k(-\nu) \frac{\chi^{n-2}(\nu)}{(n-2)!} \bar{\chi}(\nu), \quad (12)$$

determined by the NLO corrections. The series representation is one of the infinitely many possible ways, equivalent with NLA accuracy, to represent the total cross section. It has the advantage to make manifest the BFKL resummation of leading and subleading energy logarithms and is very practical in numerical computations. The well-known feature of the large NLA BFKL corrections is revealed by being the d_n coefficients of opposite sign with respect to the b_n and increasing with n in absolute value.

The neglect of NLO corrections, except for μ_R - and s_0 -dependent terms, to the impact factors affects the value of the d_n coefficients. In the case of the $\gamma^* \gamma^* \rightarrow VV$ process, it turned out that the contribution to d_n from the kernel starts to dominate over that from the impact factors for $n \geq 4$. This makes evident the fact that the high-energy behavior of the amplitude is weakly affected by the NLO corrections to the impact factor. Therefore, our approximated $\gamma^* \gamma^*$ total cross section should compare better and better with the correct result as the energy increases.

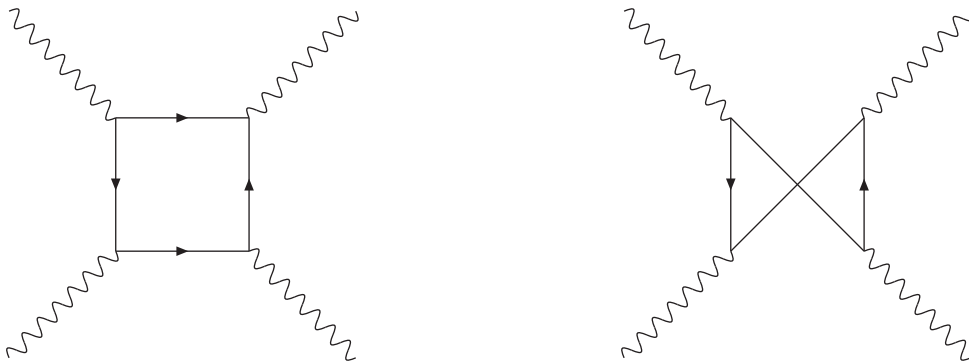


Figure 2: Quark box LO diagrams.

Our aim in this paper is to study the dependence of the cross section given in Eqs. (8)-(9) on the center-of-mass energy, both in the range of LEP2, where experimental data are available [10, 11], and of the future linear colliders. In order to stabilize the perturbative series, it is necessary to resort to some optimization procedure, exploiting the freedom to vary the energy parameters, μ_R and s_0 , without corrupting the calculation but at the next-to-NLA. Following Refs. [17], we use both the principle of minimal sensitivity (PMS method) [25] and the Brodsky-Lepage-Mackenzie (BLM) method [26]: for some selected values of the energy s in the region of interest the optimal scales μ_R and s_0 are found and the cross section is thus determined. Then, the curve giving the cross section *vs.* the energy is obtained by interpolation.

In order to compare the theoretical prediction with the existing data from LEP2, we cannot neglect the contribution from LO quark box diagrams shown in Fig. 2, which is of order $\alpha^2(\ln s)/s$. We take this contribution from Ref. [1]. To be definite, we take the so-called ϕ -averaged $\gamma^*\gamma^*$ total (hadronic) cross section, written as

$$\sigma_{QBOX}^{\gamma^*\gamma^*}(s, Q_1, Q_2) = \sum_q e_q^4 \left(\sigma_{TT} + 2\sigma_{TS} + \sigma_{SS} \right), \quad (13)$$

with the σ_{ik} , $i, k = T, S$ given in the Appendix E of Ref. [1] and evaluated for massless u -, d -, c -, s -quarks and massive b - and t -quarks. On the other hand, the soft Pomeron contribution, if estimated within the vector-dominance model, is proportional to $\sigma_{\gamma^*\gamma^*} \sim (m_V^2/Q^2)^4 \sigma_{\gamma\gamma}$ and is therefore suppressed for highly virtual photons. In the following analysis we neglect such higher twist contributions.

3 Numerical results

We restrict ourselves to the case of symmetric kinematics, which means equal virtuality $Q_1 = Q_2 \equiv Q$ for the two photons. This is the so-called “pure BFKL regime”, as opposite to the “DGLAP regime” realized for strongly ordered photon virtualities.

Let us start with the PMS optimization method [25]. Setting $Y \equiv \ln(s/Q^2)$ and $Y_0 \equiv \ln(s_0/Q^2)$, we require that, for each value of Y , the cross section given in Eqs. (8)-(9) is the least sensitive to the variation of the scales μ_R and Y_0 . For a first analysis we choose $Q^2=17 \text{ GeV}^2$ ($n_f=4$), in order to compare with the experimental data from CERN LEP2 collected for $\langle Q^2 \rangle=16 \text{ GeV}^2$ (L3) and $\langle Q^2 \rangle=18 \text{ GeV}^2$ (OPAL). We have found that the cross section is quite stable under variation of the two scales and generally exhibits only one stationary point (local maximum). In the few cases when the cross section presented more than one stationary point, we choose as optimal parameters those corresponding to the smoothest stationary point. The typical values that optimize the cross section turned out to be $\mu_R \simeq 3Q$ and $Y_0 \simeq 2$. Note that for the $\gamma^*\gamma^* \rightarrow VV$ amplitude at $Q^2=24 \text{ GeV}^2$ ($n_f=5$) the same procedure led to optimal values for μ_R as large as $\sim 10Q$.

The other optimization procedure we considered is inspired by the BLM method [26]: we perform a finite renormalization to the momentum (MOM) scheme with $\xi = 0$ and then choose the renormalization scale in order to remove the β_0 -dependent part in the cross section. The renormalization is defined as follows:

$$\alpha_s \rightarrow \alpha_s \left[1 + T_{MOM}(\xi = 0) \frac{\alpha_s}{\pi} \right], \quad T_{MOM}(\xi = 0) = T_{MOM}^{conf} + T_{MOM}^\beta,$$

$$T_{MOM}^{conf} = \frac{N_c}{8} \frac{17}{2} I, \quad T_{MOM}^\beta = -\frac{\beta_0}{2} \left[1 + \frac{2}{3} I \right], \quad I \simeq 2.3439.$$

Then, following [17], for each value of Y we found the pairs (Y_0, μ_R) for which the term proportional to β_0 in the renormalized cross section vanishes. Then, among the resulting pairs, we determined the optimal one according to the PMS principle. The typical values of the scales found in this way are very similar to those obtained with the other method.

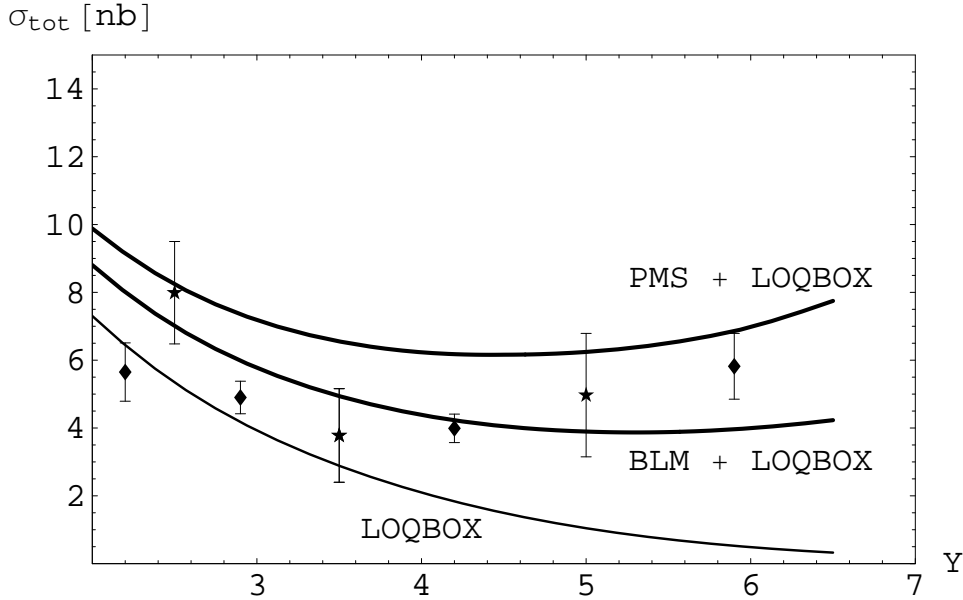


Figure 3: Energy dependence of the total cross section for the collision of two photons with virtualities $Q^2=17 \text{ GeV}^2$ ($n_f=4$) as predicted by the PMS and BLM methods, with the inclusion of the LO quark box contribution. For comparison, experimental data from OPAL [11] (stars, $Q^2=18 \text{ GeV}^2$) and L3 [10] (diamonds, $Q^2=16 \text{ GeV}^2$) are shown.

We stress that this way of using the BLM optimization method is somewhat different from Ref. [8], since there the $\gamma^*\gamma^*$ total cross section was built using a Pomeron intercept optimized by the BLM method (see Ref. [9]). Here we apply the BLM optimization procedure to the cross section itself, which is a well-defined physical quantity, while the perturbative Pomeron intercept can not be derived directly from experiment data.

In this work we consider two regions of energy: the CERN LEP2 region and a higher energy region, possibly reachable in future linear colliders. In the first of these regions we can compare our results with LEP2 experimental data and with the determinations of Ref. [8]. We admit that in this region the neglect of NLO corrections to the impact factors can play an important role; since these corrections are negative (see, for instance, Ref. [27]), our prediction will certainly overestimate the true NLO result. In the second energy region considered, we expect the role of NLO corrections to the impact factor be less relevant and, therefore, our prediction closer to the true NLA BFKL result.

In Fig. 3 we summarize our results for the CERN LEP2 region: we show the NLA BFKL curves obtained by the PMS and the BLM methods, to which we added the contribution of the LO quark box diagrams. For comparison we put in this plot also the experimental data from CERN LEP2, namely three data points from OPAL [11] ($Q^2=18 \text{ GeV}^2$) and four data points from L3 [10] ($Q^2=16 \text{ GeV}^2$). We observe first of all that the difference between the two theoretical curves can be taken as an estimate of the systematics effects which underlay the optimization procedures adopted here. The fact that the PMS curve is systematically above the BLM curve is not surprising, since the stationary point for the amplitude in the space of the parameters Y_0 and μ_R is always a local maximum, sometimes

an absolute maximum, for varying Y . The comparison with experiments shows that the PMS curve overestimates data, while the BLM curve seems to be more in agreement with them. However, if we recall that in both cases a negative contribution from NLO impact factors is being missed, it seems that the PMS curve has better chances to agree with data in the fully NLA BFKL calculation. Around the energy for which the applicability condition for the BFKL resummation, $\bar{\alpha}_s Y \sim 1$, is satisfied, which in the considered kinematics corresponds to $Y \sim 5$, *both* the PMS and BLM curves agree with experimental data within the (large) errors. Finally, we remark that the determination from Ref. [8] falls between our two curves from PMS and BLM methods. It is also important to note the the high-energy rise of the data for the cross section cannot be described only by the LO quark-box, nor can it be explained by the NLO quark box contribution [3] (the L3 datum at $Y = 6$ is underestimated by ~ 4 standard deviations, see Ref. [8]).

From the energy dependence of the NLO BFKL cross section determined through the PMS method at $Q^2=17 \text{ GeV}^2$ we obtained also the *effective* intercept (minus 1) as a function of the energy. The result is shown in Fig. 4: it turns out that the intercept grows monotonically in the energy range considered, in an approximately linear manner. In particular, at small Y the dynamical intercept is negative since quark box dominates and its energy behavior mimics a subleading Reggeon; at large Y the perturbative Pomeron is visible; around $Y \sim 4.5$ the transition between the two regimes takes place.

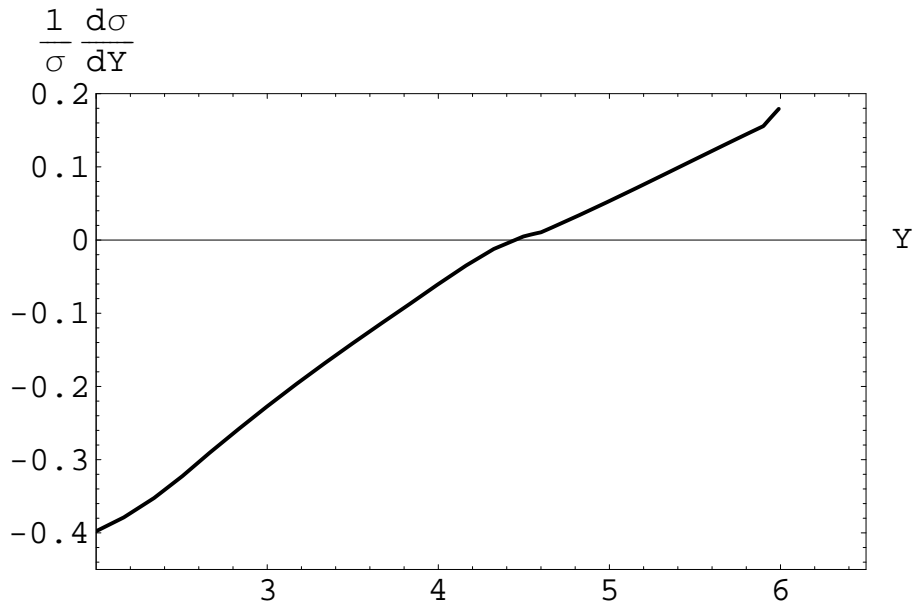


Figure 4: Energy dependence of the Pomeron intercept (minus 1) calculated from the total cross section with the PMS method at $Q^2=17 \text{ GeV}^2$ and $n_f = 4$.

In Fig. 5 we show the Y -behavior of the total cross section for $Q^2=20 \text{ GeV}^2$ ($n_f=5$) in an energy region not explored by past and present experiments, but relevant for future colliders. We plot here the two curves obtained in the present work with the PMS and the BLM methods. The applicability condition for the BFKL resummation, $\bar{\alpha}_s Y \sim 1$, corresponds here to $Y \sim 6$; around this energy the deviation between the PMS and the BLM methods is about 40%. This discrepancy can be taken as an estimate of the systematic

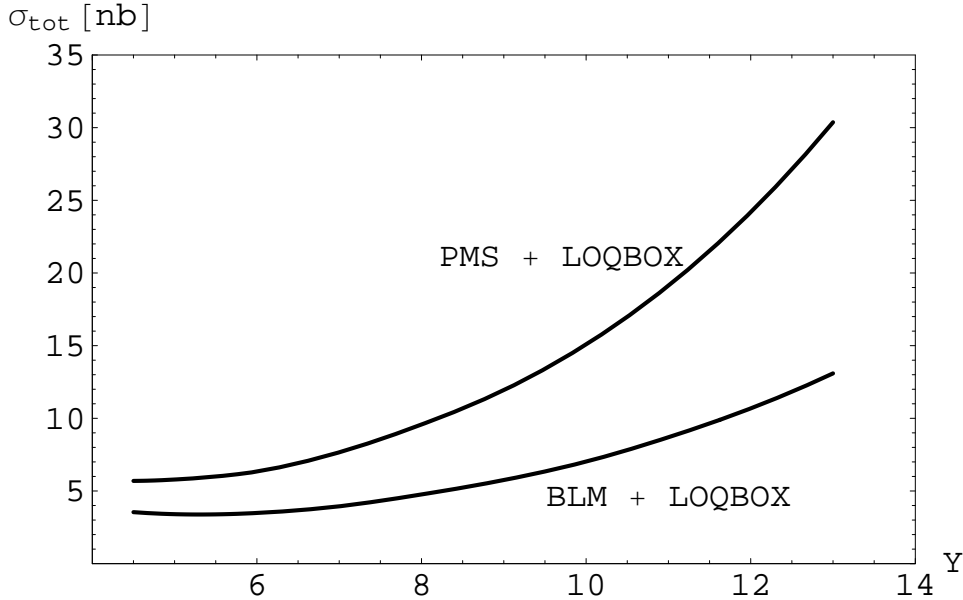


Figure 5: Energy dependence of the total cross section for the collision of two photons with virtualities $Q^2=20 \text{ GeV}^2$ ($n_f=5$) as predicted by the PMS and BLM methods.

uncertainty of this approach. We observe that our determination from the BLM method is in quite good agreement with the result of Ref. [8] (see Fig. 4 of that paper), obtained for the same kinematics. The optimal values of the energy scales in the PMS method are similar to those obtained in the kinematics region studied before ($Q^2=17 \text{ GeV}^2$) for the lower energies, with a tendency to increase for the higher energies considered.

4 Conclusions

In this paper we have presented an estimate of the energy dependence of the $\gamma^*\gamma^*$ total hadronic cross section in an energy range which covers LEP2 and future linear colliders. We have used the singlet forward BFKL Green's function at the next-to-leading order together with forward $\gamma^* \rightarrow \gamma^*$ impact factors *at the leading order*. However, we included in the impact factors the subleading terms required by the invariance of the full amplitude at the NLA under change of the renormalization scale μ_R and of the energy scale s_0 entering the BFKL approach.

We have found that, in spite of the presence of very large NLA corrections, if suitable methods are used to stabilize the perturbative series, a smooth curve for the energy behavior of the cross section can be achieved. We have considered two energy regions: the CERN LEP2 region and a region possibly reachable by future linear colliders.

Our result in the CERN LEP2 region compares favorably with experimental data. Systematic effects coming from the optimization procedure are estimated by the comparison with two different methods. We stress, however, that in this energy region the role of the

neglected NLO corrections to the impact factors can be relevant; in particular, being these corrections negative, our prediction certainly overestimates the true NLO result. Our findings in the CERN LEP2 region are in agreement with the result of Ref. [8], where for the first time subleading BFKL effects were considered in the $\gamma^*\gamma^*$ total hadronic cross section. Our calculation profits by the experience accumulated in the last few years with the application of the BFKL approach at subleading level in the case of the electroproduction of two light vector mesons [17, 18]. It can be considered as a refinement of the analysis of Ref. [8], since we apply perturbative series optimization procedures directly to the process cross section.

The numerical effect of the neglected subleading corrections to the impact factors cannot be quantified. We expected that it be modest in the second region of energy considered in this work. Here we believe that our prediction from the PMS method should be very close to the complete NLA BFKL result. The final word will be said when the $\gamma^*\gamma^*$ cross section will be calculated fully in the next-to-leading approximation.

Acknowledgment

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